OCCASIONAL PAPER 126 MARCH 1952

# SHORTCUTS FOR CRUISERS AND SCALERS

L. R. Grosenbaugh

SOUTHERN FOREST EXPERIMENT STATION
Harold L. Mitchell, Director
New Orleans, La.
FOREST SERVICE, U.S. DEPT. OF AGRICULTURE

# SHORTCUTS FOR CRUISERS AND SCALERS

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Southern Forest Experiment Station

This paper explains how to:

Cruise timber without plot boundaries or tree diameters (page 2)

Simplify much mental arithmetic (page 9)

Compute individual tree volumes rapidly without referring to volume tables (page 10)

Compute log volumes by any of four popular log rules (page 11)

Compute cull percents for logs independently of log rules (page 14)

Compute net log scale by machine accumulation of recorded net lengths and diameters without looking up individual log values (page 16)

Design sampling plans for cruising, scaling, or tree measurement (page 20)

These methods also provide a basis for converting aggregate log or tree volumes from one log rule to another, using only aggregate length or aggregate product of length times diameter.

The last technique--random sampling design--has been thoroughly covered by other authors, but it has been put into capsule form here.

## PLOTLESS TIMBER CRUISING

The following pages describe a new and very convenient way of cruising timber. The method makes it unnecessary to measure or guess tree diameter, plot radius, or strip width; and it greatly simplifies office calculations. The cruiser merely stands at the center of the plot, looks through a simple hand-held angle-gauge, and counts all trees that appear larger than the angle laid out by the gauge.

Although based on a centuries-old concept, the use of an angle-gauge to accept or reject sample trees is new, the inspiration of a European.1/ The first American exposition of the theory, published recently, adapted the method to American units of measure and developed several new applications.2/ Readers should consult it if they desire a more comprehensive account than can be given here.

For simplicity this paper assumes that the areas cruised do not slope more than 10 percent, but instrumental or computational corrections can be made to adapt the method to steeper country.

### Instrument

Before starting any field work, the cruiser will need to make an angle-gauge for optically defining an angle of 104.18 minutes with its vertex at his eye.

His simplest course is to get a 33-inch stick and mount a peephole at one end with a line of sight bisecting a 1-inch metal crosspiece at the other end. The crosspiece, when viewed through the peephole, should exactly cover a 3-foot horizontal intercept at a distance of 99 feet from the eye. The distance between crosspiece and peephole should be adjusted till this occurs. Hypsometer and Biltmore graduations may be put on the stick if desired.

A 4- to 7-power monocular with two vertical reticule lines 30.3 mils apart constitutes a better instrument. Like the crossarm device, it can be checked against a 3-foot intercept at 99 feet, except that the vertex of the 104.18-minute angle will be 1 focal length in front of the objective lens instead of at the eye. Instruments adjusting for slope are feasible, but will not be discussed here.

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1/ Bitterlich, W. Die Winkelzählprobe. Allgemeine Forst- und Holzwirtschaftliche Zeitung 59 (1/2): 4-5. 1948.
2/ Grosenbaugh, L. R. Plotless timber estimates--new, fast, easy. Journal of Forestry 50:32-37. 1952.

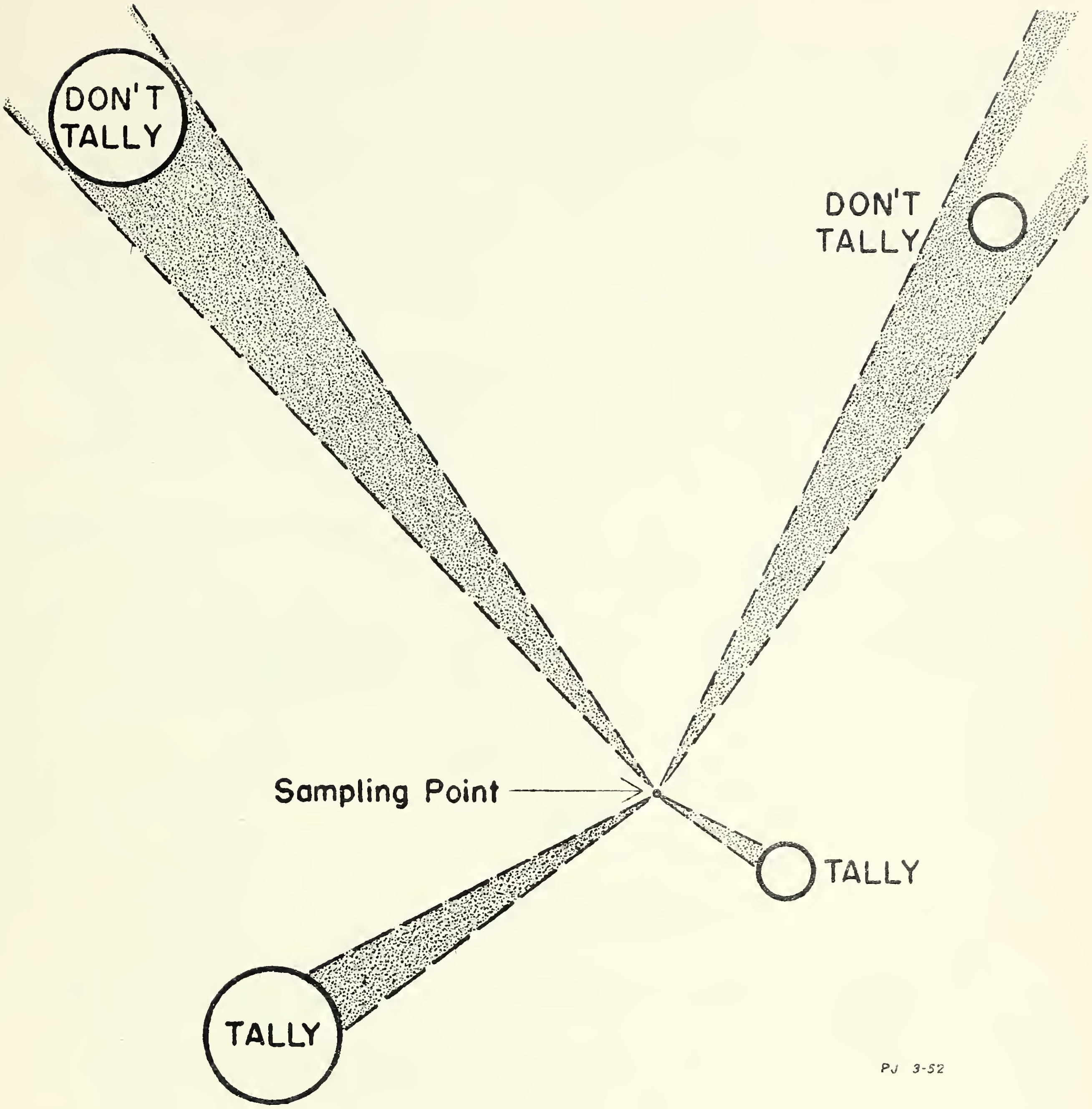



Figure 1.--Plotless timber cruising. Shaded areas represent 104.18-minute angles optically established by the instrument described on the opposite page. Circles represent cross-sections (at breast height) of trees viewed from the sampling point. Cruiser merely stands at the sampling point (analagous to a plot center), counts every tree whose d.b.h. appears larger than the angle, and disregards every tree whose d.b.h. appears smaller. All trees visible from the sampling point must be counted or rejected. The count of trees, multiplied by 10, gives an estimate of basal area per acre. In the diagram, only two trees are counted, so the basal area estimate is 20 square feet per acre. Reliable estimates require more than one sample, of course.

## Basal Area Estimates

Assured of an instrument, the cruiser should decide on the pattern of sampling points (analagous to plot centers) that he wishes to employ on the area to be cruised. He must then visit each sampling point (or at least an unbiased point in its vicinity), look in every direction through his instrument, and count the number of trees whose d.b.h.'s appear larger than the crosspiece. The principle is illustrated in figure 1. The eyepiece (or vertex) of the angle-gauge should pivot on the sampling point until the count is completed, except that it may be temporarily moved sideways perpendicular to the line of sight to clear nearby brush or trees likely to mask other qualifying trees. After a little practice, the cruiser will find he can gauge all but borderline trees by eye alone.

Suppose that the cruiser has tallied a total of 240 qualifying trees at 30 unbiased sampling points on an area.

$$\text{Estimated basal area per acre} = (10)\left(\frac{\text{Number of tallied trees}}{\text{Number of sampling points}}\right)$$

$$= (10)\left(\frac{240}{30}\right)$$

$$= 80 \text{ sq. ft.}$$

## Pulpwood Estimates

Suppose, further, that the stand on the area is pine of pulpwood size in two even-aged groups. Qualifying trees would be counted in the same manner as above, but each age group should be tallied separately and the average total height of each group should be recorded. For example, 100 tallied trees might have had an average total height of 50 feet, while 140 tallied trees might have had an average total height of 80 feet, assuming the same 30 sampling points were used.

$$\text{Estimated rough stacked volume per acre} = \frac{\sum(\text{Number of tallied trees})(\text{Av. ht. in ft.})}{(20)\ (\text{Number of sampling points})}$$

$$= \frac{(100)(50) + (140)(80)}{(20)(30)}$$

$$= \frac{16{,}200}{600}$$

$$= 27 \text{ standard cords}$$

Local form, utilization, and stacking practice may justify the use of some divisor other than 20. Sampling stacked volume per square foot of basal area in various height classes will determine the divisor to use. It will usually be between 18 and 21.

Greater precision is possible where merchantable length of each qualifying tree can be predicted with some degree of confidence. The same 240 trees at 30 sampling points might have been tallied according to their expected merchantable lengths, thus:

50 trees with 30 lineal feet of pulpwood
50 trees with 40 lineal feet of pulpwood
50 trees with 50 lineal feet of pulpwood
50 trees with 60 lineal feet of pulpwood
40 trees with 70 lineal feet of pulpwood
Total: 240 trees tallied at 30 sampling points

Cubic volume per acre (including bark) could be estimated by multiplying the tally for each length class (shown in column 3 below) by the appropriate $\frac{\text{cubic volume}}{\text{basal area}}$ factors (shown in column 2 below). The sum of these products is used to obtain cubic feet as indicated below. Of course, local $\frac{\text{cubic volume}}{\text{basal area}}$ factors for each merchantable length class would be best, but the ready-made factors will serve wherever great accuracy is unnecessary.

Example:

| Merch. length class (ft.) | Cu. vol. factor (incl. bark) | Tally | Product of factor x tally |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 10 | 7 | 0 | 0 |
| 20 | 14 | 0 | 0 |
| 30 | 20 | 50 | 1,000 |
| 40 | 26 | 50 | 1,300 |
| 50 | 31 | 50 | 1,550 |
| 60 | 36 | 50 | 1,800 |
| 70 | 39 | 40 | 1,560 |
| | Total: | 240 trees; | 7,210 = sum of products |

$$\text{Estimated cubic volume per acre (incl. bark)} = (10)\left(\frac{\text{Sum of products}}{\text{Number of sampling points}}\right)$$

$$= (10)\left(\frac{7,210}{30}\right)$$

$$= 2,403 \text{ cubic feet}$$

Cubic feet can be divided by a locally appropriate conversion divisor to give rough stacked cords per acre (26.7 cords if local divisor were 90 cubic feet per cord).

## Miscellaneous Estimates

The number of trees tallied by the angle-gauge in any given class tends to be directly proportional to the total basal area per acre in that class. This fact allows estimating basal area distribution by species, quality, vigor, diameter, length, or any other desired criterion, by merely classifying tallied trees.

Basal area in each merchantable length class may be easily converted to various units of volume, because $\frac{\text{volume}}{\text{basal area}}$ ratios within merchantable length classes are relatively stable, regardless of diameter. For simplicity, it will be assumed that 40 trees of the same species have been tallied at 5 sampling points and also classified as to their merchantable length in terms of 16-foot sawlogs, thus:

8 trees with zero logs
10 trees with one log
11 trees with two logs
7 trees with three logs
4 trees with four logs

Total: 40 trees tallied at 5 sampling points

It can be quickly estimated by earlier formulae that stand basal area per acre is $(10)\left(\frac{40}{5}\right) = 80$ square feet, of which 16 square feet is in zero-log trees, 20 is in one-log trees, 22 in two-log trees, 14 in three-log trees, and 8 in four-log trees.

Volume per acre in terms of International rule (1/4-inch kerf), Scribner rule, Doyle rule, or peeled cubic feet (without bark) could be estimated by multiplying tally in each merchantable length class by appropriate factor tabled below, summing the products, and proceeding as indicated.

| Merch. length class | Volume factors: Int. ¼-inch | Scribner | Doyle | Peeled cubic | Example: Tally | Product of peeled cubic factor x tally |
|---|---|---|---|---|---|---|
| Zero-log | 0 | 0 | 0 | 0 | 8 | 0 |
| One-log | 7 | 6 | 4 | 12 | 10 | 120 |
| Two-log | 13 | 11 | 8 | 20 | 11 | 220 |
| Three-log | 18 | 16 | 12 | 27 | 7 | 189 |
| Four-log | 23 | 20 | 15 | 34 | 4 | 136 |
| Five-log | 28 | 25 | 21 | 40 | 0 | 0 |
| | | | | Total: | 40 trees; | 665 = sum of products |

$$\text{Estimated peeled cubic volume per acre} = (10)\left(\frac{\text{Sum of products}}{\text{Number of sampling points}}\right)$$

$$= (10)\left(\frac{665}{5}\right)$$

$$= 1{,}330 \text{ cubic feet}$$

$$\text{Estimated board-foot volume per acre} = (100)\left(\frac{\text{Sum of appropriate products}}{\text{Number of sampling points}}\right)$$

= 8,620 bd. ft. Int. ¼-inch

= 7,460 bd. ft. Scribner

= 5,440 bd. ft. Doyle

The ready-made volume factors given above will satisfy the accuracy needs of many cruisers, but it is possible to attain any desired precision by merely sampling local $\frac{\text{volume}}{\text{basal area}}$ ratios for each merchantable length class recognized (1/10 of that ratio has been used in case of board-foot factors).

## Variable Plot Radius or Point-Sampling Concept

The 104.18-minute angle-gauge tallies all trees closer to the sampling point than 33 times tree d.b.h. Whatever the size of the tallied tree, the instrument guarantees that it is within a plot radius determined by that particular tree's d.b.h.

Thus, a tree with a d.b.h. of 1/2 foot (and a basal area of 1/5 square foot) would be tallied inside a plot with a radius of 16-1/2 feet (and an area of 1/50 acre). The basal area of each 6-inch tree must, then, be multiplied by 50 to place it on a per-acre basis. This means that such a tree would contribute $(50)\left(\frac{1}{5}\right) = 10$ square feet to the estimate of basal area per acre. However, a tree with a d.b.h. of 1 foot (and a basal area of 4/5 square foot) would be tallied inside a plot with a radius of 33 feet (and an area of 4/50 acre). This means that each 12-inch tree would contribute $\left(\frac{50}{4}\right)\left(\frac{4}{5}\right) = 10$ square feet to the estimate of basal area per acre. Tallied trees of different sizes would have different basal areas, different plot areas, and hence different blow-up factors, but the product of each tallied tree basal area times its appropriate blow-up factor would always be 10 square feet--the constant contribution of any tallied tree to the estimate of basal area per acre. This is why diameter of a tallied tree and its distance from the sampling point are immaterial.

Some people can conceive the idea more clearly by visualizing a sheet of paper with a known number of evenly spaced sample points plotted on it. On the sheet are placed 2 transparent disks of different size, each exhibiting on its face a small concentric circle with diameter 1/66 that of the disk, and with area $\left(\frac{1}{66}\right)^2$ or $\left(\frac{1}{4356}\right)$ that of the disk. Next, the number of disks sampled at each plotted point is counted (it will be 0, 1, or 2 in this simple example) and the counts at all points are summed. Those familiar with dot-counting know that $\frac{\text{Sum of disk counts}}{\text{Number of sample points}}$ estimates $\frac{\text{Area of disks}}{\text{Area of sheet}}$, and $\left(\frac{1}{4356}\right)\left(\frac{\text{Sum of disk counts}}{\text{Number of sample points}}\right)$ estimates $\frac{\text{Area of small circles}}{\text{Area of sheet}}$. Multiplying this last by 43,560 square feet (one acre) estimates the square feet of small circles per acre of sheet or $(10)\left(\frac{\text{Sum of disk counts}}{\text{Number of sample points}}\right)$, which explains the reasoning behind computations on page 4.

In cruising, the sheet of paper becomes the land to be cruised, small circles become tree cross-sections, and disks become surrounding zones within which a sampling point must fall in order for the 104.18-minute angle-gauge to accept that particular tree for tally.

## MENTAL ARITHMETIC

By memorizing the squares of numbers 1 through 25, a scaler or cruiser can greatly facilitate the ordinary volume calculations he performs mentally. The squares of numbers from 26 through 125 can be easily obtained from the first twenty-five, and multiplication can frequently be simplified.

<u>To square numbers 26 through 75</u>: subtract 25 from the number, suffix 2 zeros, and add the square of (50 minus the number).

Example: $(36)^2 = (36-25)00 + (50-36)^2 = 1{,}100 + (14)^2 = 1{,}296$
$(63)^2 = (63-25)00 + (50-63)^2 = 3{,}800 + (13)^2 = 3{,}969$

<u>To square numbers 76 through 125</u>: subtract 100 from twice the number, suffix 2 zeros, and add the square of (100 minus the number).

Example: $(83)^2=(166-100)00 + (100-83)^2=6{,}600 + 289 = 6{,}889$
$(111)^2=(222-100)00 + (100-111)^2=12{,}200 + 121 = 12{,}321$

<u>To square numbers ending in $\frac{1}{2}$</u>: square the whole portion, add the whole portion, and add $\frac{1}{4}$ (this last step can usually be ignored).

Example: $(25\frac{1}{2})^2 = 625 + 25 + \frac{1}{4} = 650\frac{1}{4}$

<u>To multiply numbers</u>: square the smaller, and add the smaller times the difference (useful where numbers differ by no more than 10).

Example: $(27)(36) = 729 + 243 = 972$

Alternatively, take $\frac{1}{4}$ the difference between the square of the sum and the square of the difference; or else take the difference between the square of the half-sum and the square of the half-difference (plus the smallest of the original numbers if fractions are dropped prior to squaring).

Example: $(27)(36) = \frac{(63)^2-(9)^2}{4} = \frac{3{,}969-81}{4} = \frac{3{,}888}{4} = 972$

$(27)(36) = (31)^2-(4)^2 + 27 = 961-16+27 = 972$

<u>Squares to be memorized</u>

| N | $N^2$ | N | $N^2$ | N | $N^2$ |
|---|---|---|---|---|---|
| 1 | 1 | 11 | 121 | 21 | 441 |
| 2 | 4 | 12 | 144 | 22 | 484 |
| 3 | 9 | 13 | 169 | 23 | 529 |
| 4 | 16 | 14 | 196 | 24 | 576 |
| 5 | 25 | 15 | 225 | 25 | 625 |
| 6 | 36 | 16 | 256 | | |
| 7 | 49 | 17 | 289 | | |
| 8 | 64 | 18 | 324 | | |
| 9 | 81 | 19 | 361 | | |
| 10 | 100 | 20 | 400 | | |

## USEFUL NEW RULES FOR TREE VOLUME

The author has devised 6 simple rules for tree volume which can be easily memorized and mentally calculated for individual trees; 3 of them can be used to convert aggregate International $\frac{1}{4}$-inch board-foot volumes to other scales without reworking cruise data.

In all six rules:

D stands for d.b.h. (outside bark) in inches.
N stands for number of 16-foot logs to a merchantable sawlog top.
M stands for number of 10-foot sticks to a merchantable pulpwood top.

Saw timber

Rule 1: Board feet (Int. $\frac{1}{4}$") $= \frac{(D-N)^2(N)}{2}$ where D lies between 6 and 50

Rule 2: Cubic feet (peeled) $= \frac{\text{Int.}\frac{1}{4}\text{"bd.ft.}+2ND}{9}$ where D lies between 6 and 50

Rule 3: Board feet Scribner $= \text{Int.}\frac{1}{4}\text{"bd.ft.}-\frac{ND}{2}$ where D lies between 6 and 50

Rule 4: Board feet Doyle $= \text{Int.}\frac{1}{4}\text{"bd.ft.}-32N$ where D lies between 11 and 32

Pulpwood

Rule 5: Cubic feet (including bark) $= \frac{(D+2)^2(M)}{40}$ where M lies between 0 and 8

Rule 6: Number of trees per rough stacked cord $= \frac{3600}{(D+2)^2(M)}$ where M lies between 0 and 8

Experience will indicate when 5, 10, or 15 percent adjustments are needed to allow for peculiarities of local utilization or taper differing from Girard Form Class 80 in the case of sawtimber or Girard Form Point (91-D) in the case of pulpwood. In the last case, local experience may indicate factors other than 40 or 3600 which will be more accurate.

If the sum of ND and the sum of N are accumulated on tally sheets (along with International $\frac{1}{4}$-inch board-foot volumes according to locally valid volume tables), rules 2, 3, and 4 allow easy conversion to other units, without need for developing different local volume tables or working up each tree volume anew.

## POPULAR RULES FOR LOG VOLUME

Legend for all rules:

- $d$ = average diameter of log inside bark at small end (inches)
- $D$ = average diameter of log inside bark at large end (inches)
- $L$ = length of log (feet)
- $K$ = constant (2 for paraboloid, 3 for conoid, 4 for subneiloid)

General solid of revolution:

$$\text{Volume (cu. ft.)} = .005454154\, L \left[ Dd + \frac{(D-d)^2}{K} \right]$$

Conoid tapering 1 inch per 8 lineal feet:

$$\text{Volume (cu. ft.)} = .005454154\, Ld^2 + .0006817692 L^2 d + .00002840705 L^3$$

$$\text{Surface (sq. ft.)} = .2618029 Ld + .01636268 L^2$$

International log rule with taper allowance of ½ inch per 4 lineal feet (master formulae):

| With 1/8-inch kerf | | With 1/4-inch kerf (=.904762 value for 1/8"kerf) | |
|---|---|---|---|
| + .05500000 | $Ld^2$ | + .04976191 | $Ld^2$ |
| + .006875000 | $L^2d$ | + .006220239 | $L^2d$ |
| − .2050000 | $Ld$ | − .1854762 | $Ld$ |
| + .00028645833 | $L^3$ | + .0002591767 | $L^3$ |
| − .01281250 | $L^2$ | − .01159226 | $L^2$ |
| + .04666667 | $L$ | + .04222222 | $L$ |
| Volume (bd. ft.) = sum of 6 terms | | Volume (bd. ft.) = sum of 6 terms | |

Coefficients for the International 1/8-inch rule are accurate to an infinite number of digits (if repeating decimals be extended), since they have been obtained by use of numerical progression summation formulae. Coefficients for the International ¼-inch rule and the conoidal rules have 7 significant digits.

Traditionally, International rule formulae have been applied only to lengths which are multiples of 4 feet, with linear interpolation for intervening values. Interpolated values are trivially higher than exact formulae values.

Final International rule values have usually been rounded to nearest 5 board feet.

Formulae for given diameters and varying lengths may be easily derived from the master formulae and will contain only 3 terms. Formulae for given lengths and varying diameters may also be obtained with only 3 terms, as illustrated below. When used for deriving a formula for a given length, each coefficient below should be multiplied by the desired length to eliminate L from all terms, but the special form given below will be needed when accumulating length and diameter.

Coefficients for volume of logs of average length by log rules allowing taper

| Average log length in feet | Coefficients for board feet (Int. $\frac{1}{4}$-inch) | | | Coefficients for cubic feet (conoid, 1 inch per 8 feet) | | |
|---|---|---|---|---|---|---|
| | $Ld^2$ | Ld | L | $Ld^2$ | Ld | L |
| 4 | +.0498 | -.161 | .000 | +.00545 | +.0027 | +.0005 |
| 8 | " | -.136 | -.034 | " | +.0055 | +.0018 |
| 12 | " | -.111 | -.060 | " | +.0082 | +.0041 |
| 16 | " | -.086 | -.077 | " | +.0109 | +.0073 |
| 20 | " | -.061 | -.086 | " | +.0136 | +.0114 |
| 24 | " | -.036 | -.087 | " | +.0164 | +.0164 |
| 28 | " | -.011 | -.079 | " | +.0191 | +.0223 |
| 32 | " | +.014 | -.063 | " | +.0218 | +.0291 |
| 36 | " | +.038 | -.039 | " | +.0245 | +.0368 |
| 40 | " | +.063 | -.007 | " | +.0273 | +.0455 |
| 44 | " | +.088 | +.034 | " | +.0300 | +.0550 |
| 48 | " | +.113 | +.083 | " | +.0327 | +.0654 |

Coefficients for volume of logs of any length by log rules ignoring taper

| Average log length in feet | Coefficients for board feet Doyle | | | Coefficients for board feet Scribner (curved) | | |
|---|---|---|---|---|---|---|
| | $Ld^2$ | Ld | L | $Ld^2$ | Ld | L |
| Any | +.0625 | -.500 | +1.000 | +.0494 | -.124 | -.269 |

Volumes of 16-foot logs may be approximated adequately by 4 rules of thumb:

$$\text{Int. } \tfrac{1}{4}\text{-inch bd. ft. per 16-foot log} = .8\,(d - 1)^2$$

$$\text{Cu. ft. per 16-foot log} = \frac{(d + 1)^2}{A}$$

where A is 11 if taper tends to be slightly more than 2 inches
where A is $11\frac{1}{2}$ if taper is 2 inches and linear
where A is 12 if taper tends to be slightly less than 2 inches

$$\text{Scribner bd. ft. per 16-foot log} = .8\,(d - 1)^2 - \frac{d}{2}$$

$$\text{Doyle bd. ft. per 16-foot log} = (d - 4)^2$$

## CULL PERCENT

Defect reduces the amount of lumber otherwise recoverable from a log. Most log rules are biased in their estimates of lumber yielded by a sound log, and the bias usually varies with log diameter. It would be logical to estimate the proportion of lumber lost through defect by a technique unaffected by the inconsistencies of badly biased log rules. Such a proportion could easily be translated into log-rule board feet by multiplying by the gross scale according to any desired rule, or it could be translated into equivalent cull length by multiplying by log length. This procedure is traditional where entire sections, wedge-like sectors, or sweepy lunes are lost, but interior defects have long been inconsistently boxed and computed in terms of mill tally. The author has devised the more universally consistent method described in (5) below. For convenience, formulae for cull proportion caused by four other common defects are also outlined. None of the five formulae depend on log scale. In all five, d is average diameter of log at small end in inches and L is length in feet.

(1) Proportion lost when defect affects entire section:

$$\left(\frac{\text{Length of defective section}}{L}\right)$$

(2) Proportion lost when defect affects wedge-shaped sector:

$$\left(\frac{\text{Length of defective section}}{L}\right)\left(\frac{\text{central angle of defect}}{360^\circ}\right)$$

(3) Proportion lost when log sweeps (or when its curved central axis departs more than 2 inches from an imaginary chord connecting the centers of its end-areas; ignore sweep less than 2 inches):

$$\left(\frac{\text{Maximum departure minus 2 inches}}{d}\right)$$

(4) Proportion lost when log crooks (or when a relatively short section deflects abruptly from straight axis of longer portion of log):

$$\left(\frac{\text{Length of deflecting section}}{L}\right)\left(\frac{\text{maximum deflection}}{d}\right)$$

(5) Proportion lost when average cross-section of interior defect is enclosable in ellipse (or circle) with major and minor diameters measurable in inches:

$$\left(\frac{(\text{Major})(\text{minor})}{(d-1)^2}\right)\left(\frac{\text{length of defect}}{L}\right)$$

In applying rule 5, defect in peripheral inch of log (slab collar) can be ignored, but the ellipse should enclose a band of sound wood at least 1/2-inch thick. Where it is necessary to use a rectangle instead of an ellipse to enclose the defect, the cull

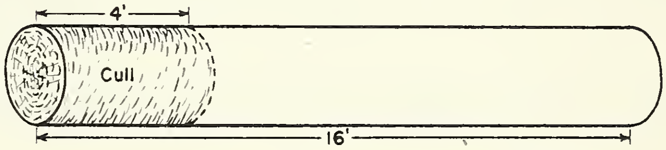
percent will be 5/4 as much as for an ellipse with the same diameters as the rectangle. An obvious modification when a ring of rot surrounds a sound heart with average diameter H (in inches) is to estimate the proportion sound as $\frac{(H-1)^2}{(d-1)^2}$ and the proportion defective as $1-\frac{(H-1)^2}{(d-1)^2}$.

In the rare case where cubic scale for other products than sawlogs is being used, sweep ordinarily is not considered to cause loss, and $(d + 1)^2$ is used instead of $(d - 1)^2$ as a divisor for interior defect deduction.

When net log scale is desired at the time of scaling, as is often the case, proportion defective should be multiplied by L, and the equivalent cull length C thus obtained can be considered as applicable to an equivalent cull section with the same scaling diameter as the log. Direct multiplication of cull percent by International gross volume will be preferable when scaling by that rule, however, unless equivalent cull lengths are to be processed as described in the following section.

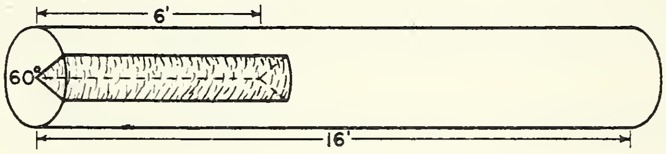
CULL SECTION (Rule 1):

Cull Percent $= \frac{4}{16} = 25\%$



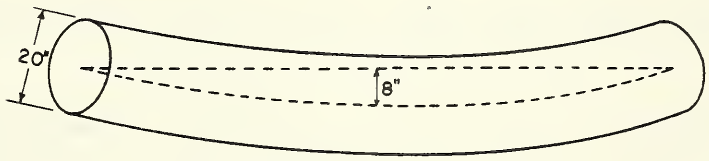

CULL SECTOR (Rule 2):

Cull Percent $= \left(\frac{6}{16}\right)\left(\frac{60}{360}\right) = 6\frac{1}{4}\%$



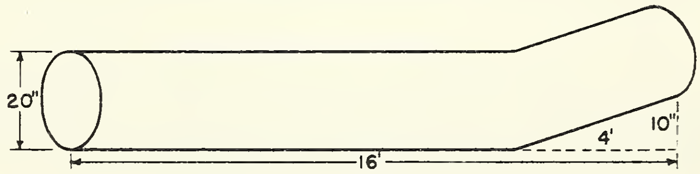

SWEEP (Rule 3):

Cull Percent $= \frac{8-2}{20} = 30\%$



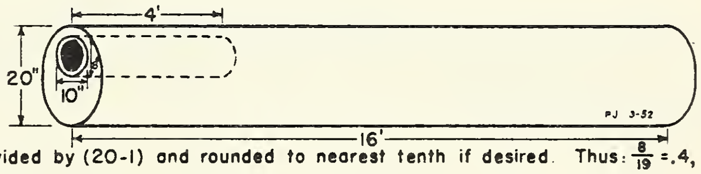

CROOK (Rule 4):

Cull Percent $= \left(\frac{10}{20}\right)\left(\frac{4}{16}\right) = 12\frac{1}{2}\%$




INTERIOR DEFECT (Rule 5):

Cull Percent $= \frac{(8)(10)}{(20-1)^2}\left(\frac{4}{16}\right) = 5\frac{5}{9}\%$




In practice each ellipse axis can be divided by (20-1) and rounded to nearest tenth if desired. Thus: $\frac{8}{19} = .4$, $\frac{10}{19} = .5$, and $(.4)(.5)\left(\frac{4}{16}\right) = 5\%$

Figure 2.--Illustration of cull percent for a log 16 feet long and 20 inches in diameter.

## LOG SCALE BY DIAMETER-LENGTH ACCUMULATIONS

Where logs are scaled by a single rule, the usual direct scaling procedure and the cull deduction discussed above are quite convenient. Where logs are to be scaled by several rules, however, or where it is desired to maximize field output and minimize field error, a better technique is available.

Log diameter (d), log length (L), and equivalent cull length (C) (or proportion cull x log length) are the only measurements recorded. Logs longer than 16 feet will ordinarily be broken down into several shorter logs, for each of which estimated d, L, C should be recorded. By the process explained below, one or two sets of 3 sums each are accumulated in the office on a calculator (any fully automatic machine such as Marchant ACR-8M, ACT-10M; Monroe CAA-10, CSA-10; or similar models).

Steps in triple accumulation on the Marchant are as follows:

1. With the calculator conveniently tabbed, set up the first d starting on the left bank of the keyboard and simultaneously set up its square ($d^2$) ending on the right bank of the keyboard.

2. With the multiplying keys, multiply the above setup of d and $d^2$ by the appropriate L.

3. Return the carriage to its original tabbed position, clearing the keyboard but not the upper or middle dials.

4. Repeat the operation, using the next d, $d^2$, and appropriate L, until every d and L has been entered.

5. The upper dial will now read $\Sigma L$; the middle dial will read $\Sigma Ld$ on the left and $\Sigma Ld^2$ on the right.

6. After the totals are recorded and the machine cleared, the previous steps are repeated for d, $d^2$, and C.

The only special precaution needed is to place a zero in front of any d, L, or C which is less than 10, so the accumulation will be correctly lined up. On the Monroe, L is set up before d and $d^2$ are, but otherwise the process is similar.

Following are examples that show how gross scales are computed from $\Sigma Ld^2$, $\Sigma Ld$, $\Sigma L$; how cull scales are computed from $\Sigma Cd^2$, $\Sigma Cd$, $\Sigma C$; or alternatively, how net scales are computed from $\Sigma (L-C)d^2$, $\Sigma (L-C)d$, $\Sigma (L-C)$.

| Entry no. | d | L | C | Description of defect |
|---|---|---|---|---|
| | Inches | Feet | Feet | |
| 1 | 12 | 16 | 4 | Log with 5" of total sweep. |
| {2 | 14 | 08 | 3} | 18' log with center rot in 8" x 10" ellipse 6' deep at small end. |
| 3} | 15 | 10 | | |
| 4 | 15 | 12 | | |
| 5 | 08 | 12 | | |
| 6 | 24 | 14 | 3 | Log with 90° cull sector 14' long. |
| {7 | 13 | 12 | } | 24' log with 2' butt section entirely cull. |
| 8} | 15 | 12 | 2} | |
| 9 | 27 | 14 | | |
| 10 | 16 | 16 | | |

N=10: $\Sigma L = 126$ | $12 = \Sigma C$

$\Sigma Ld = 2,036$ | $192 = \Sigma Cd$

$\Sigma Ld^2 = 36,684$ | $3,342 = \Sigma Cd^2$

Length of average entry $= \frac{\Sigma L}{N} = 12.6$ ft.

Coefficients for Doyle or Scribner are independent of average length and may be obtained directly from the table on page 12. Coefficients for International 1/4-inch or cubic feet must be interpolated from the first table on page 12, for average length 12.6 feet. As long as variation in L is not large (and it is kept low by breaking down long logs into several entries), interpolated coefficients will be quite accurate. Gross scales, then, by the various rules are (using 12.6-ft. average length):

Doyle $.0625\ \Sigma Ld^2 - .500\ \Sigma Ld + 1.000\ \Sigma L = 1,401$ bd. ft. gross

Scribner $.0494\ \Sigma Ld^2 - .124\ \Sigma Ld - .269\ \Sigma L = 1,526$ bd. ft. gross

Int. 1/4" $.0498\ \Sigma Ld^2 - .107\ \Sigma Ld - .063\ \Sigma L = 1,601$ bd. ft. gross

Cubic $.00545\ \Sigma Ld^2 + .0086\ \Sigma Ld + .0046\ \Sigma L = 218.0$ cu. ft. gross

Exactly the same coefficients are used on $\Sigma Cd^2$, $\Sigma Cd$, $\Sigma C$, and give the following cull for the 3 board-foot rules (cubic feet of saw-log cull could be similarly calculated if desired):

Doyle $.0625\ \Sigma Cd^2 - .500\ \Sigma Cd + 1.000\ \Sigma C = 125$ bd. ft. cull

Scribner $.0494\ \Sigma Cd^2 - .124\ \Sigma Cd - .269\ \Sigma C = 138$ bd. ft. cull

Int. 1/4" $.0498\ \Sigma Cd^2 - .107\ \Sigma Cd - .063\ \Sigma C = 145$ bd. ft. cull

Where only net scale is desired, the cull deduction can be applied to L at the time of scaling, so that d, (L - C), and C are recorded. Before obtaining the length of average entry, $\sum C$ should be added to $\sum(L - C)$. Net scale thus directly obtained will be the same as subtracting the above cull from the previously computed gross scales.

| Entry no. | d Inches | L - C Feet | C Feet |
|---|---|---|---|
| 1 | 12 | 12 | 4 |
| 2 | 14 | 05 | 3 |
| 3 | 15 | 10 | |
| 4 | 15 | 12 | |
| 5 | 08 | 12 | |
| 6 | 24 | 11 | 3 |
| 7 | 13 | 12 | |
| 8 | 15 | 10 | 2 |
| 9 | 27 | 14 | |
| 10 | 16 | 16 | |

$N = 10$; $\sum(L - C)$ 114

$\sum(L - C)d$ 1,844

$\sum(L - C)d^2 = 33{,}342$

$12 = \sum C$

$$\text{Av. gross length} = \frac{\sum(L-C) + \sum C}{N} = 12.6 \text{ ft.}$$

Coefficients for these net accumulations will be exactly the same as for the gross or cull accumulations, since the 12.6-foot average length governs in both cases where taper affects the coefficients. Net board foot scales from the net accumulations, then, are:

Doyle:

$$.0625 \sum(L-C)d^2 - .500 \sum(L-C)d + 1.000 \sum(L-C) = 1{,}276 \text{ bd. ft. net}$$

Scribner:

$$.0494 \sum(L-C)d^2 - .124 \sum(L-C)d - .269 \sum(L-C) = 1{,}388 \text{ bd. ft. net}$$

Int. $\frac{1}{4}$-inch:

$$.0498 \sum(L-C)d^2 - .107 \sum(L-C)d - .063 \sum(L-C) = 1{,}456 \text{ bd. ft. net}$$

Note that the sum of net plus cull equals previous gross, as might be expected. This is the method recommended for ordinary use where only net scale is needed.

Those who desire to develop coefficients from local mill tallies instead of using those appropriate to a given log rule will find the subject is well covered in Schumacher and Jones 3/.

Had the individual gross scales corresponding to each entry been laboriously recorded (with cull by International $\frac{1}{4}$-inch rule), the results would have differed only trivially in the third digit, if at all. Thus,

| Entry no. | d | L | Gross Doyle | Gross Scrib. Dec. C | Gross Int. $\frac{1}{4}$-in. | Cull (Int. $\frac{1}{4}$-in.) | Gross cubic 1/ |
|---|---|---|---|---|---|---|---|
| | Inches | Feet | - - - - - - Board feet | | | - - - - - - - | Cu.ft. |
| 1 | 12 | 16 | 64 | 80 | 95 | 25 | 14.8 |
| {2 | 14 | 8 | 50 | 60 | 65 | 25 | 9.2 |
| 3 | 15 | 10 | 75 | 90 | 95 | | 13.3 |
| 4 | 15 | 12 | 91 | 110 | 115 | | 16.2 |
| 5 | 08 | 12 | 12 | 20 | 25 | | 5.0 |
| 6 | 24 | 14 | 350 | 350 | 370 | 90 | 47.3 |
| {7 | 13 | 12 | 61 | 70 | 85 | | 12.4 |
| 8 | 15 | 12 | 91 | 110 | 115 | 20 | 16.2 |
| 9 | 27 | 14 | 463 | 480 | 470 | | 59.4 |
| 10 | 16 | 16 | 144 | 160 | 180 | | 25.2 |
| | | | 1,401 | 1,530 | 1,615 | 160 | 219.0 |
| | | | | | less 160 | | |
| | | | | | net 1,455 | | |

1/ Cubic volumes are for conoid with diameter increasing 1 inch per 8 lineal feet.

3/ Schumacher, F.X., and Jones, W. C., Jr. Empirical log rules and the allocation of sawing time to log size. Journal of Forestry 38: 889-896. 1940.

## RANDOM SAMPLING DESIGN

Since systematic square grids of plots or mechanically spaced lines of plots tend to give more precise estimates than the same number of random plots or random lines of plots, the random error formulae given below should be used with caution and will provide only a rough upper bound for estimates of error where systematic sampling is employed. Those wishing to estimate the error of a systematic sample more precisely are referred to DeLury 4/. It should be remembered that the limits of error discussed below will be exceeded by 1 out of 3 sample means from a normal random population. Doubling the given coefficient of variation will result in more intensive sampling with only about 1 chance in 20 that the sample mean will not lie within the desired limits of error.

Individual sampling units chosen for volume measurement may comprise a population of point-counts, plot volumes, volumes of clusters of plots, volumes of lines of plots (as in conventional cruising), strip volumes, log volumes, or tree volumes. Variation in sample volumes (X) between (N) randomly selected individual units is conveniently estimated in terms of the mean volume and expressed as:

(A) Coefficient of variation in percent =

$$(100)\left(\frac{\text{standard deviation}}{\text{mean volume}}\right) = 100\sqrt{\left(\frac{N}{N-1}\right)\left(\frac{N\Sigma X^2}{(\Sigma X)^2} - 1\right)}$$

Thus, if 5 sample-plot volumes (expressed in thousands of board feet per acre) were 2, 6, 10, 10, 12, with a mean of 8, the coefficient of variation would be:

$$100\sqrt{\left(\frac{5}{4}\right)\left(\frac{1920}{1600} - 1\right)} = 100\sqrt{.25} = 50 \text{ pct.}$$

The standard error of the mean of (N) random volumes from an infinite normal population may be estimated in terms of the mean volume and expressed in terms of the coefficient of variation:

(B) Standard error as percent of mean (infinite population) =

$$\frac{\text{Coefficient of variation in pct.}}{\sqrt{N}} = 100\sqrt{\left(\frac{1}{N-1}\right)\left(\frac{N\Sigma X^2}{(\Sigma X)^2} - 1\right)}$$

Thus, the standard error (in percent) above would be:

$$\frac{50}{\sqrt{5}} = 22.4 \text{ percent}$$

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4/ DeLury, D. B. Values and integrals of the orthogonal polynomials up to $\underline{n}$ = 26. Univ. Toronto Press, Toronto, Ont., 33 pp. 1950.

Where the population is finite, and an appreciable proportion has been sampled, the standard error in percent is further reduced, as indicated below (in an infinite population, of course, the unsampled proportion is always 1):

(C) Standard error as percent of mean (finite population) =

$$(\text{coefficient of variation in pct.})\sqrt{\frac{\text{unsampled proportion}}{N}}$$

Thus, if the previous example, composed of 5 samples, had come from a population of only 25 possible sampling units (instead of one with infinite possible sampling units) the standard error (in percent) would have been:

$$50\sqrt{\frac{.80}{5}} = (50)(.4) = 20 \text{ percent}$$

Where it is desired to design a random sample of an infinite population so that its standard error is in the neighborhood of a specified $\pm$ limit of error in percent, the following formula is useful; it is the basis for the table at the end of this section:

(D) Number of samples to be taken from infinite population = $N_\infty$

$$N_\infty = \left(\frac{\text{coefficient of variation in percent}}{\text{specified limit of error in percent}}\right)^2$$

Thus, if an infinite population of plot volumes had a coefficient of variation of 50 percent, and it were desired to sample it randomly with a design whose standard error would be in the neighborhood of $\pm$ 10 percent, the number of random samples to select would be: $\left(\frac{50}{10}\right)^2 = 25$. This same result could have been read from the table at the end of this section.

Where the population is finite (composed of M individuals), $N_\infty$ calculated by the previous formula must be replaced by N, as follows:

(E) $$N = \frac{1}{\frac{1}{N_\infty} + \frac{1}{M}} = \frac{1}{\left(\frac{\text{specified limit of error in percent}}{\text{coefficient of variation in percent}}\right)^2 + \frac{1}{M}}$$

Thus, if there had been only 25 possible individual plot volumes (instead of an infinite number) in the previous example, it would not have been necessary to sample all 25 to approach a $\pm$ 10 percent standard error. Instead, the number of samples to be taken would have been:

$$\frac{1}{\frac{1}{25} + \frac{1}{25}} = 12\text{-}1/2 \text{ (or 13)}$$

The above formula is easily converted to the so-called "sampling fraction" (or proportion of the finite population to be sampled) by dividing by the number in the population (M):

(F) Proportion of finite population to be sampled = $\frac{N}{M}$

$$\frac{N}{M} = \frac{1}{\frac{M}{N_\infty} + 1} = \frac{N_\infty}{N_\infty + M}$$

Thus, in the previous example, the proportion of the population to be sampled (where coefficient of variation is 50 percent, specified limit of error is $\pm$ 10 percent, and finite population contains 25 individuals) would be: $\frac{1}{\frac{25}{25} + 1} = \frac{1}{2}$. This means that 1 out of every 2 individuals in the population must be sampled.

It is frequently advantageous to stratify a finite population into groups or classes of homogeneous individuals according to some criterion other than actually sampled volume. Where reasonably similar reduced coefficients of variation prevail for several groups, it is convenient and not too inappropriate to consider the largest of the reduced values as applying to the whole population thus stratified. With optimum allocation of sampling to strata, formula (F) would apply to the population as a whole, although the proportions would vary according to volume in each stratum. However, it is usually convenient to adopt a constant or uniform sampling fraction, and to dispense with the correction for finite population as a margin of safety, thus:

(G) Approximate proportion of finite stratified population to be sampled = $\frac{N_\infty}{M}$.

(where coefficients of variation of strata tend to be similar)

Thus, if approximately 10,000 pine trees were to be counted and classified into 2-inch groups having within-strata coefficients of variation of 30 percent, and it was desired to sample individual tree volumes with a standard error approaching 1-1/2 percent, the proportion to be sampled would be $\frac{400}{10,000} = \frac{1}{25}$ using the table below. Sampling 1 tree for volume out of every 25 counted should be accomplished in some unbiased manner.

As a rough guide to approximate coefficients of variation, the figures given below may be useful in the South in the absence of valid local information:

| Individual sample | Population | Random coefficient of variation Percent |
|---|---|---|
| 1/5-acre plot volume | Finite unstratified plots | 100 |
| Point-count (104.18' angle) | Infinite unstratified points | 50 |
| Tree volume | Finite pine trees, stratified in 2-inch groups | 30 |
| Tree volume | Finite pine trees, stratified in 4-inch groups | 35 |
| Tree volume | Finite pine trees, stratified in 6-inch groups | 40 |
| Tree volume | Finite hardwood trees, stratified in 2-inch groups | 35 |
| Tree volume | Finite hardwood trees, stratified in 4-inch groups | 40 |
| Tree volume | Finite hardwood trees, stratified in 6-inch groups | 50 |

The following table gives $N_\infty$ (the number of samples needed from an infinite population with a given coefficient of variation) so that the standard error of the samples will tend to be in the neighborhood of a given limit of error.

Number of samples to be taken from infinite population = $N_\infty$

| Coefficient of variation (percent) | Specified percent limit for standard error | | | |
|---|---|---|---|---|
| | ± $1\frac{1}{2}$ percent | ± 5 percent | ± 10 percent | ± 20 percent |
| | $N_\infty$ | | | |
| 5 | 12 | 1 | 1 | 1 |
| 10 | 45 | 4 | 1 | 1 |
| 15 | 100 | 9 | 3 | 1 |
| 20 | 178 | 16 | 4 | 1 |
| 25 | 278 | 25 | 7 | 2 |
| 30 | 400 | 36 | 9 | 3 |
| 35 | 545 | 49 | 13 | 4 |
| 40 | 712 | 64 | 16 | 4 |
| 45 | 900 | 81 | 21 | 6 |
| 50 | 1,112 | 100 | 25 | 7 |
| 55 | 1,345 | 121 | 31 | 8 |
| 60 | 1,600 | 144 | 36 | 9 |
| 65 | 1,878 | 169 | 43 | 11 |
| 70 | 2,178 | 196 | 49 | 13 |
| 75 | 2,500 | 225 | 57 | 15 |
| 80 | 2,845 | 256 | 64 | 16 |
| 85 | 3,212 | 289 | 73 | 19 |
| 90 | 3,600 | 324 | 81 | 21 |
| 95 | 4,012 | 361 | 91 | 23 |
| 100 | 4,445 | 400 | 100 | 25 |
| 125 | 6,945 | 625 | 157 | 40 |
| 150 | 10,000 | 900 | 225 | 57 |
| 175 | 13,612 | 1,225 | 307 | 77 |
| 200 | 17,778 | 1,600 | 400 | 100 |